Continuation of the dual amplitude with Mandelstam analyticity off mass shell

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Abstract

The off mass shell continuation of dual amplitude with Mandelstam analyticity (DAMA) is proposed. The modified DAMA (M-DAMA) preserves all the attractive properties of DAMA, such as its pole structure and Regge asymptotics, and leads to a generalized dual amplitude $A(s,t,Q^2)$. In such a way we complete a unified "two-dimensionally dual" picture of strong interaction [1, 2, 3, 4]. This generalized amplitude can be checked in the known kinematical limits, i.e. it should reduce to the ordinary dual amplitude on mass shell, and to the nuclear structure function when $t=0$. We fix the Q^2-dependence in M-DAMA by comparing the structure function F_2, resulting from it, with phenomenological parameterizations. The results of M-DAMA are in qualitative agreement with the experiment in all studied regions, i.e. in the large and low x limits as well as in the resonance region.

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1 Introduction

This work is devoted to modeling of the scattering amplitude for inelastic electron-proton scattering. The kinematics of inclusive ep scattering, applicable to both high energies, typical of HERA, and low energies as at JLab, is shown in Fig. 1. We introduce virtuality Q^2, $Q^2 = -q^2 = -(k-k')^2 \geq 0$, and Bjorken variable $x = Q^2/2p \cdot q$. These variables x, Q^2 and Mandelstam variable s (of the $\gamma^* p$ system), $s = (p+q)^2$, obey the relation:

$$s = Q^2(1-x)/x + m^2\,, \tag{1}$$

where m is the proton mass. And Fig. 2 shows how inelastic $\gamma^* p$ scattering is related to the forward elastic (t=0) $\gamma^* p$ scattering, and then the latter is decomposed into a sum of the $s-$channel resonance exchanges.

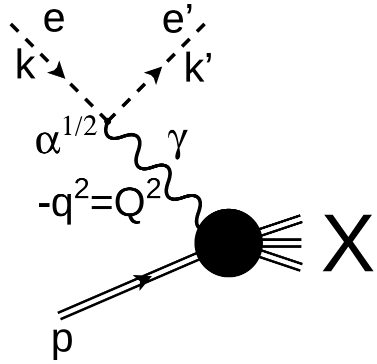

Figure 1: Kinematics of inelastic electron-proton scattering.

About thirty years ago Bloom and Gilman [5] observed that the prominent resonances in inelastic $e^- p$ scattering (see Fig. 1) do not disappear with increasing photon virtuality Q^2, but fall at roughly the same rate as background. Furthermore, the smooth scaling limit proved to be an accurate average over resonance bumps seen at lower Q^2 and s, this is so called Bloom-Gilman or hadron-parton duality. Since the discovery, the hadron-parton duality was studied in a number of papers [6] and the new supporting data has come from the recent experiments [7, 8]. These studies were aimed mainly to answer the

questions: in which way a limited number of resonances can reproduce the smooth scaling behaviour? The main theoretical tools in these studies were finite energy sum rules and perturbative QCD calculations, whenever applicable. Our aim instead is the construction of an explicit dual model combining direct channel resonances, Regge behaviour typical for hadrons and scaling behaviour typical for the partonic picture. Some attempts in this direction have already been done in Refs. [1, 2, 3, 4], which we will discuss in more details below.

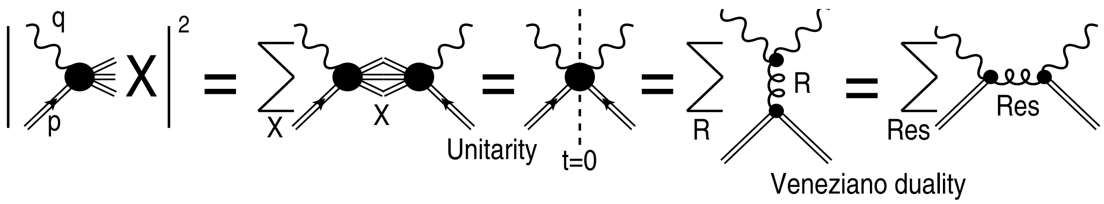

Figure 2: According to the Veneziano (or resonance-reggeon) duality a proper sum of either t-channel or s-channel resonance exchanges accounts for the whole amplitude. From [2].

The possibility that a limited (small) number of resonances can build up the smooth Regge behaviour was demonstrated by means of finite energy sum rules [9]. Later it was confused by the presence of an infinite number of narrow resonances in the Veneziano model [10], which made its phenomenological application difficult, if not impossible. Similar to the case of the resonance-reggeon duality [9], the hadron-parton duality was established [5] by means of the finite energy sum rules, but it was not realized explicitly like the Veneziano model (or its further modifications).

First attempts to combine resonance (Regge) behaviour with Bjorken scaling were made [11, 12, 13] at low energies (large x), with the emphasis on the right choice of the Q^2-dependence, such as to satisfy the required behaviour of form factors, vector meson dominance (the validity (or failure) of the (generalized) vector meson dominance is still disputable) with the requirement of Bjorken scaling. Similar attempts in the high-energy (low x) region became popular recently stimulated by the HERA data. These are discussed in section 3.

Recently in a series of papers [1, 2, 3, 4] authors made attempts to build a generalized Q^2-dependent dual amplitude $A(s,t,Q^2)$. This amplitude, a function of three variables, should have correct known limits, i.e. it should reduce to the on shell hadronic scattering amplitude on mass shell, and to the nuclear structure function (SF) when $t = 0$. In such a way we could complete a unified "two-dimensionally dual" picture of strong interaction [1, 2, 3, 4] - see Fig. 3.

In Ref. [1, 2] the authors tried to introduce Q^2-dependence in Veneziano amplitude [10] or more advanced Dual Amplitude with Mandelstam Analyticity (DAMA) [14]. The Q^2-dependence can be introduced either through a Q^2-dependent Regge trajectory [1], leading to a problem of physical interpretation of such an object, or through the g parameter of DAMA [1, 2]. This last way seems to be more realistic [2], but it is allowed only in the limited range of Q^2 due to the DAMA model requirement $g > 1$ [14] (see [2] for details).

In the papers [3, 4] the authors went in an opposite direction - they built a Regge-dual model with Q^2-dependent form factors, inspired by the pole series expansion of DAMA, which fits the SF data in the resonance region[1]. The hope was to reconstruct later the Q^2-dependent dual amplitude, which would lead to such an expansion.

A consistent treatment of the problem requires the account for the spin dependence, which we ignore in this paper for the sake of simplicity. Our goal is rather to check qualitatively the proposed new way of constructing the "two-dimensionally dual" amplitude.

[1]It is important that DAMA not only allows, but rather requires nonlinear complex Regge trajectories [14]. Then the trajectory with restricted real part lead to a limited number of resonances.

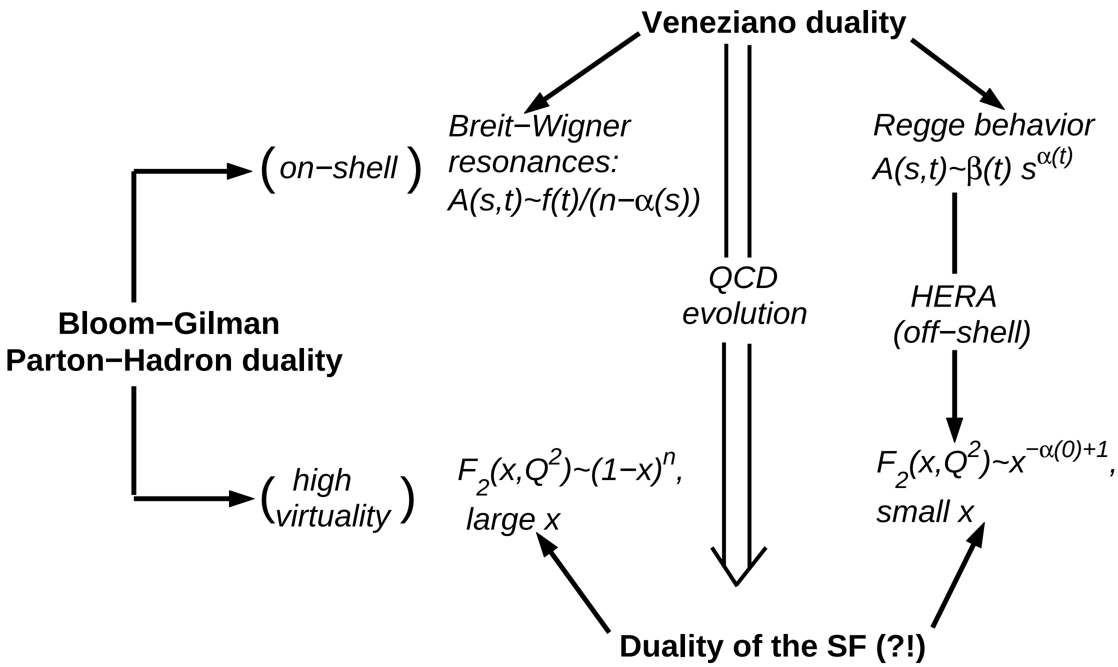

Figure 3: Veneziano, or resonance-reggeon duality [10] and Bloom-Gilman, or hadron-parton duality [5] in strong interactions. From [2].

2 Modified DAMA model

The DAMA integral is a generalization of the integral representation of the B-function used in the Veneziano model [14] [2]:

$$D(s,t) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s')-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(t'')-1}, \tag{2}$$

where $a' = a(1-z)$, $a'' = az$, and g is a free parameter, $g > 1$, and $\alpha_s(s)$ and $\alpha_t(t)$ stand for the Regge trajectories in the $s-$ and $t-$channels.

In this paper we propose a modified definition of DAMA (M-DAMA) with Q^2-dependence [15]. It also can be considered as a next step in generalization of the Veneziano model. M-DAMA preserves all the attractive features of DAMA, such as pole decompositions in s and t, Regge asymptotics etc., yet it gains the Q^2-dependent form factors, correct large and low x behaviour for $t = 0$ etc.

The proposed M-DAMA integral reads [15]:

$$D(s,t,Q^2) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s')-\beta(Q^{2''})-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(t'')-\beta(Q^{2'})-1}, \tag{3}$$

where $\beta(Q^2)$ is a smooth dimensionless function of Q^2, which will be specified later on from studying different regimes of the above integral.

The on mass shell limit, $Q^2 = 0$, leads to the shift of the $s-$ and $t-$channel trajectories by a constant factor $\beta(0)$ (to be determined later), which can be simply absorbed by the trajectories and, thus, M-DAMA reduces to DAMA. In the general case of the virtual particle with mass M we have to replace Q^2 by $(Q^2 + M^2)$ in the M-DAMA integral.

[2]There are several integral representations of DAMA [14], here we shall use the most common one.

Now all the machinery developed for the DAMA model (see for example [14]) can be applied to the M-DAMA integral. Below we shall report briefly only some of its properties, relevant for the further discussion.

2.1 Singularities in M-DAMA

The dual amplitude $D(s,t,Q^2)$ is defined by the integral (3) in the domain $\mathcal{R}e\ (\alpha_s(s')+\beta(Q^{2''}))<0$ and $\mathcal{R}e\ (\alpha_t(t'')+\beta(Q^{2'}))<0$. For monotonically decreasing function $\mathcal{R}e\ \beta(Q^2)$ (or non-monotonic function with maximum at $Q^2=0$) and for increasing or constant real parts of the trajectories these equations, applied for $0\le z\le 1$, mean $\mathcal{R}e\ (\alpha_s(s)+\beta(0))<0$ and $\mathcal{R}e\ (\alpha_t(t)+\beta(0))<0$. To enable us to study the properties of M-DAMA in the domains $\mathcal{R}e\ (\alpha_s(s')+\beta(Q^{2''}))\ge 0$ and $\mathcal{R}e\ (\alpha_t(t'')+\beta(Q^{2'}))\ge 0$, which are of the main interest, we have to make an analytical continuation of M-DAMA. This leads to the appearance of two moving poles

$$\begin{aligned}\alpha_s(s(1-z_n))+\beta(Q^2 z_n)&=n \quad \text{and}\\ \alpha_t(tz_m)+\beta(Q^2(1-z_m))&=m, \quad n,m=0,1,2...\end{aligned} \tag{4}$$

The singularities of the dual amplitude are generated by pinches which occur in the collisions of the above mentioned moving and fixed singularities of the integrand $z=0,1$.

1. The collision of a moving pole $z=z_n$ with the branch point $z=0$ results in a pole at $s=s_n$, where s_n is defined by
$$\alpha_s(s_n)+\beta(0)=n\,. \tag{5}$$
Please, notice the presence of an extra (in comparison to DAMA) term $\beta(0)$. It can be considered as a shift of the trajectory. If $\beta(0)$ is an integer number, then the modification is trivial.

2. The collision of a moving pole $z=z_n$ with the branch point $z=1$ results in a pole at $Q^2=Q^2_n$, defined by
$$\alpha_s(0)+\beta(Q^2_n)=n\,. \tag{6}$$
In this sense we can think about $\beta(Q^2)$ as of a kind of trajectory, but we do not mean that it describes real physical particles. Also we will see later that with a proper choice of $\beta(Q^2)$ we can avoid these unphysical poles, and $\beta(Q^2)$ required by the low x behaviour of the nucleon SF is exactly of this type.

3. Similarly, the collision of a moving pole $z=z_m$ with the branch point $z=1$ results in a pole at $t=t_m$, defined by
$$\alpha_t(t_m)+\beta(0)=m\,. \tag{7}$$

4. The collision of a moving pole $z=z_m$ with the branch point $z=0$ results in a pole at $Q^2=Q^2_m$, defined by
$$\alpha_t(0)+\beta(Q^2_m)=m\,. \tag{8}$$
Note that if $\alpha_s(0)=\alpha_t(0)$ the poles in Q^2 will be degenerate. For further discussion we shall consider a non-degenerated case.

2.2 Pole decompositions

Similarly as for DAMA [14], case 1 from the above results into pole decomposition of M-DAMA amplitude with the following expression for the pole term [15]:

$$D_{s_n}(s,t,Q^2)=g^{n+1}\sum_{l=0}^{n}\frac{[\beta'(0)Q^2-s\alpha'_s(s)]^l C_{n-l}(t,Q^2)}{[n-\alpha_s(s)-\beta(0)]^{l+1}}\,, \tag{9}$$

where

$$C_l(t,Q^2) = \frac{1}{l!}\frac{d^l}{dz^l}\left[\left(\frac{1-z}{g}\right)^{-\alpha_t(tz)-\beta(Q^2(1-z))-1}\right]_{z=0} . \tag{10}$$

Formula (9) shows that our $D(s,t,Q^2)$ does not contain ancestors and that an $(n+1)$-fold pole emerge on the n-th level. The crossing-symmetric term can be obtained in a similar way by considering the case 3 from the list above.

The modifications with respect to DAMA are A) the shift of the trajectory $\alpha_s(s)$ by the constant factor of $\beta(0)$ (we can easily remove this shift including $\beta(0)$ into trajectory); B) the coefficients C_l are now Q^2-dependent and can be directly associated with the form factors. The presence of the multipoles, eq. (9), does not contradict the theoretical postulates. On the other hand, they can be removed without any harm to the dual model by means the so-called Van der Corput neutralizer[3]. This procedure [14] seems to work for M-DAMA equally well as for DAMA and will result in a "Veneziano-like" pole structure:

$$D_{s_n}(s,t,Q^2) = g^{n+1}\frac{C_n(t,Q^2)}{n-\alpha_s(s)-\beta(0)} . \tag{11}$$

The Q^2-pole terms can be obtained by considering cases 2 and 4 from section 2.1, but, as we shall see later in section 4, with our choice of $\beta(Q^2)$ we avoid Q^2 poles.

2.3 Asymptotic properties of M-DAMA

Let us now discuss the asymptotic properties of M-DAMA. Using exactly the same method as in [14] it is possible to show that if the trajectory satisfies some restriction on its increase, then we obtain the Regge asymptotic behaviour [15]:

$$D(s,t,Q^2) \sim s^{\alpha_t(t)+\beta(0)} g^{\beta(Q^2)} , \quad s\to\infty . \tag{12}$$

So, in the Regge limit M-DAMA has the same asymptotic behaviour as DAMA (except for the shift $\beta(0)$).

It is more interesting to study the new regime, which does not exist in DAMA - the limit $Q^2\to\infty$, with constant s, t. We assume that $\beta(Q^2)\to-\infty$ for $Q^2\to\infty$. Then [15],

$$D(s,t,Q^2)|_{Q^2\to\infty} \approx (2g)^{2\beta(Q^2/2)+\alpha_s(s/2)+\alpha_t(t/2)+2}\sqrt{\frac{2\pi}{W}} , \tag{13}$$

where $W \approx 8\gamma\ln(Q^2/Q_0^2)$. For DIS, as we shall see below, if s and t are fixed and $Q^2\to\infty$ then $u=-2Q^2\to-\infty$, as it follows from the kinematic relation $s+t+u=2m^2-2Q^2$. So, we need also to study the $D(u,t,Q^2)$ term in this limit. If $|\alpha_u(-2Q^2)|$ is growing slower than $|\beta(Q^2)|$ or terminates when $Q^2\to\infty$, then the previous result (eq. (13), s to be changed to $u=-2Q^2$) is still valid.

3 Nucleon structure function

The total cross section of γ^*p scattering is related to the SF by

$$F_2(x,Q^2) = \frac{Q^2(1-x)}{4\pi\alpha(1+4m^2x^2/Q^2)}\sigma_t^{\gamma^*p} , \tag{14}$$

[3]In brief, the procedure [14] is to multiply the integrand of (3) by a function $\phi(z)$, which has the following properties:

$$\phi(0)=0, \quad \phi(1)=1, \quad \phi^n(1)=0, \quad n=1,2,3,...$$

The function $\phi(z)=1-exp\left(-\frac{z}{1-z}\right)$, for example, satisfies the above conditions.

where α is the fine structure constant. In eq. (14) we neglected $R(x,Q^2) = \sigma_L(x,Q^2)/\sigma_T(x,Q^2)$, which is a reasonable approximation.

The total cross section is related to the imaginary part of the scattering amplitude

$$\sigma_t^{\gamma^* p}(x,Q^2) = \frac{8\pi}{P_{CM}\sqrt{s}} \mathcal{I}m\ A(s(x,Q^2), t=0, Q^2)\ . \tag{15}$$

where P_{CM} is the center of mass momentum of the reaction, $P_{CM} = \frac{s-m^2}{2(1-x)}\sqrt{\frac{1+4m^2x^2/Q^2}{s}}$ for DIS. Thus, we have

$$F_2(x,Q^2) = \frac{4Q^2(1-x)^2}{\alpha\,(s-m^2)\,(1+4m^2x^2/Q^2)^{3/2}} \mathcal{I}m\ A(s(x,Q^2), t=0, Q^2)\,. \tag{16}$$

The minimal model for the scattering amplitude is a sum [17]

$$A(s,0,Q^2) = c(s-u)(D(s,0,Q^2) - D(u,0,Q^2)), \tag{17}$$

providing the correct signature at high-energy limit, where c is a normalization coefficient. As it was said at the beginning, we disregard the symmetry properties of the problem (spin and isospin), concentrating on its dynamics.

In the low x limit: $x \to 0$, $t = 0$, $Q^2 = const$, $s = Q^2/x \to \infty$, $u = -s$ we obtain from eqs. (12,17):

$$\mathcal{I}m\ A(s,0,Q^2)|_{s\to\infty} \sim s^{\alpha_t(t)+\beta(0)+1} g^{\beta(Q^2)}\,. \tag{18}$$

Our philosophy in this section is the following: we specify a particular choice of $\beta(Q^2)$ in the low x limit and then we use M-DAMA integral (3) to calculate the dual amplitude, and correspondingly SF, in all kinematical domains. We will see that the resulting SF has qualitatively correct behaviour in all regions. Even more - our choice of $\beta(Q^2)$ will automatically remove Q^2 poles.

According to the two-component duality picture [18], both the scattering amplitude A and the structure function F_2 are the sums of the diffractive and non-diffractive terms. At high energies both terms are of the Regge type. For $\gamma^* p$ scattering only the positive-signature exchanges are allowed. The dominant ones are the Pomeron and f Reggeon, respectively. The relevant scattering amplitude is as follows:

$$B(s,Q^2) = iR_k(Q^2)\Big(\frac{s}{m^2}\Big)^{\alpha_k(0)}, \tag{19}$$

where α_k and R_k are Regge trajectories and residues and k stands either for the Pomeron or for the Reggeon. The residue is chosen to satisfy approximate Bjorken scaling for the SF [19, 20]. From eqs. (16,19) SF is given as:

$$F_2(x,Q^2) \sim Q^2 R_k(Q^2)\Big(\frac{s}{m^2}\Big)^{\alpha_k(0)-1}\,. \tag{20}$$

Bjorken variable $x = Q^2/s$ for $s \to \infty$ and thus, Regge asymptotics and scaling behaviour require that

$$R_k(Q^2) \sim (Q^2)^{-\alpha_k(0)}\,. \tag{21}$$

Actually, it could be more involved if we require the correct $Q^2 \to 0$ limit to be respected and the observed scaling violation (the "HERA effect") to be included. Various models to cope with the above requirements have been suggested [16, 19, 20]. At HERA, especially at large Q^2, scaling is so badly violated that it may not be explicit anymore.

In the phenomenological models which are used nowadays to fit F_2 data [19, 20, 7, 8, 24] (also [3, 4] were discussed in introduction) the Q^2-dependence is introduced "by hands", via residue in the form (21), parameters of which are then fitted to the data. Now we have a model which contains Q^2-dependence from the very beginning and automatically gives a correct behaviour of the residues.

Data show that the Pomeron exchange leads to a rising structure function at large s (low x). To provide for this we have two options: either to assume supercritical Pomeron with $\alpha_P(0) > 1$ or to assume a critical ($\alpha_P(0) = 1$) dipole (or higher multipole) Pomeron [16, 21, 22]. The latter leads to the logarithmic behaviour of the SF:

$$F_{2,P}(x,Q^2) \sim Q^2 R_P(Q^2) \ln\Big(\frac{s}{m^2}\Big)\,, \tag{22}$$

which proves to be equally efficient [16, 22].

Let us now come back to M-DAMA results. Using eqs. (16,18) we obtain:

$$F_2 \sim s^{\alpha_t(0)+\beta(0)} Q^2 g^{\beta(Q^2)}\,. \tag{23}$$

Choosing

$$\beta(0) = -1 \tag{24}$$

we restore the asymptotics (20) and this allows us to use trajectories in their commonly used form. Now we have to find such a $\beta(Q^2)$, which can provide for Bjorken scaling. If we choose $\beta(Q^2)$ in the form

$$\beta(Q^2) = d - \gamma \ln(Q^2/Q_0^2)\,, \tag{25}$$

with

$$\gamma = (\alpha_t(0) + \beta(0) + 1)/\ln g = \alpha_t(0)/\ln g\,, \tag{26}$$

where d, Q_0^2 are some parameters, we get the exact Bjorken scaling.

Actually, the expression (25) might cause problems in the $Q^2 \to 0$ limit. To avoid this, it is better to use a modified expressions

$$\beta(Q^2) = \beta(0) - \gamma \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right) = -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right)\,. \tag{27}$$

This choice leads to

$$F_2(x,Q^2) \sim x^{1-\alpha_t(0)} \Big(\frac{Q^2}{Q^2+Q_0^2}\Big)^{\alpha_t(0)}\,, \tag{28}$$

where slowly varying factor $\left(\frac{Q^2}{Q^2+Q_0^2}\right)^{\alpha_t(0)}$ is typical for the Bjorken scaling violation (for example [20]).

Now let us turn to the large x limit. In this regime $x \to 1$, s is fixed, $Q^2 = \frac{s-m^2}{1-x} \to \infty$ and correspondingly $u = -2Q^2$. Using eqs. (13,16,17) we obtain:

$$F_2 \sim (1-x)^2 Q^4 g^{2\beta(Q^2/2)} \sqrt{\frac{2\pi}{W}} \left(g^{\alpha_s(s/2)} - g^{\alpha_u(-Q^2)}\right)\,. \tag{29}$$

For $Q^2 \to \infty$ factors $\left(g^{\alpha_s(s/2)} - g^{\alpha_u(-Q^2)}\right)$ and W are slowly varying functions of Q^2 under our assumption about $\alpha_u(-Q^2)$. Thus, we end up with qualitatively correct behaviour

$$F_2 \sim \left(\frac{2Q_0^2}{Q^2}\right)^{2\gamma \ln 2g} \sim (1-x)^{2\alpha_t(0)\ln 2g/\ln g}\,. \tag{30}$$

Let us now study F_2 given by M-DAMA in the resonance region. The existence of resonances in SF at large x is not surprising by itself: as it follows from (15) and (16) they are the same as in $\gamma^* p$ total cross section, but in a different coordinate system.

For M-DAMA the resonances in s-channel are defined by the condition (5). For simplicity let us assume that we performed the Van der Corput neutralization and, thus, the pole terms appear in the

form (11). In the vicinity of the resonance $s = s_{Res}$ only the resonance term $D_{Res}(s, 0, Q^2)$ is important in the scattering amplitude and correspondingly in the SF.

Using $\beta(Q^2)$ in the form (27), which gives Bjorken scaling at large s, we obtain from eq. (10):

$$C_1(Q^2) = \left(\frac{gQ_0^2}{Q^2+Q_0^2}\right)^{\alpha_t(0)} \left[\alpha_t(0) + \ln g \frac{Q^2}{Q^2+Q_0^2} - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right)\right]. \tag{31}$$

The term $\left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{\alpha_t(0)}$ gives the typical Q^2-dependence for the form factor (the rest is a slowly varying function of Q^2).

If we calculate higher orders of C_n for subleading resonances, we will see that the Q^2-dependence is still defined by the same factor $\left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{\alpha_t(0)}$. Here comes the important difference from the Regge-dual model [3, 4] motivated by introducing Q^2-dependence through the parameter g. As we see from eq. (11), g enters with different powers for different resonances on one trajectory - the powers are increasing with the step 2. Thus, if $g \sim \left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{\triangle}$, then the form factor for the first resonance is $(n = 0) \sim \left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{\triangle}$, and for the second one $(n = 2)$ it is $\sim \left(\frac{Q_0^2}{Q^2+Q_0^2}\right)^{3\triangle}$ etc. As discussed in [4] the present accuracy of the data does not allow to discriminate between the constant powers of form factor (for example Refs. [23, 7, 8, 24], and this work) and increasing ones.

4 How to avoid Q^2 poles?

General study of the M-DAMA integral allows the existence of Q^2 poles (see cases 2, 4 in section 2.1) which would be unphysical. The appearance and properties of these singularities depend on the particular choice of the function $\beta(Q^2)$, and for our choice, given by eq. (27), the Q^2 poles can be avoided.

We have chosen $\beta(Q^2)$ to be a decreasing function, then, according to conditions (6,8), there are no Q^2 poles in M-DAMA in the physical domain $Q^2 \geq 0$, if

$$\mathcal{R}e\ \beta(0) < -\alpha_s(0), \quad \mathcal{R}e\ \beta(0) < -\alpha_t(0). \tag{32}$$

We have already fixed $\beta(0) = -1$, eq. (24), and, thus, we see that indeed we do not have Q^2 poles, except for the case of supercritical Pomeron with the intercept $\alpha_P(0) > 1$. Such a supercritical Pomeron would generate one unphysical pole at $Q^2 = Q^2_{pole}$ defined by equation

$$-1 - \frac{\alpha_P(0)}{\ln g} \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right) + \alpha_P(0) = 0 \quad \Rightarrow \quad Q^2_{pole} = Q_0^2(g^{\frac{\alpha_P(0)-1}{\alpha_P(0)}} - 1). \tag{33}$$

Therefore we can conclude that M-DAMA does not allow a supercritical trajectory - what is good property from the theoretical point of view, since such a trajectory violates the Froissart-Martin limit [25].

As it was discussed above there are other phenomenological models which use dipole Pomeron with the intercept $\alpha_P(0) = 1$ and also fit the data (see for example [16]). This is a very interesting case - ($\alpha_t(0) = 1$) - for the proposed model. At the first glance it seems that we should anyway have a pole at $Q^2 = 0$. It should result from the collision of the moving pole $z = z_0$ with the branch point $z = 0$, where $\alpha_t(0) + \beta(Q^2(1 - z_0)) = 0$ in our case. Then, checking the conditions for such a collision:

$$\alpha_t(0) - t\,\alpha_t'(0)z_0 + \beta(Q^2) - \beta'(Q^2)Q^2 z_0 = 0 \;\Rightarrow\; z_0 = \frac{-\alpha_t(0) - \beta(Q^2)}{t\,\alpha_t'(0) - Q^2\beta'(Q^2)},$$

we see that for $t = 0$ and for $\beta(Q^2)$ given by eq. (27) the collision is simply impossible, because $z_0(Q^2)$ does not tend to 0 for $Q^2 \to 0$. Thus, for the Pomeron with $\alpha_P(0) = 1$ M-DAMA does not contain any unphysical singularity.

On the other hand, a Pomeron trajectory with $\alpha_P(0) = 1$ does not produce rising SF (20), as required by the experiment. So, we need a harder singularity and the simplest one is a dipole Pomeron. A dipole Pomeron produces poles of the second power - $D_{dipole}(s, t_m) \propto \frac{C(s)}{(m-\alpha_P(t)+1)^2}$, see for example ref. [21] and references therein. Formally such a dipole Pomeron can be written as $\frac{\partial}{\partial \alpha_P} \frac{C(s)}{(m-\alpha_P(t)+1)}$, and generalizing this - $D_{dipole}(s,t) = \frac{\partial}{\partial \alpha_P} D(s,t)$, where $D(s,t)$ can be given for example by DAMA or M-DAMA. Applying this expression to the asymptotic formula of M-DAMA, eq. (12), we obtain a term $g^{\beta(Q^2)} s^{\alpha_t(t)+\beta(0)} \ln s$, which then leads to a logarithmically rising SF (for $\alpha_P(0) + \beta(0) = 0$) - the one given by eq. (22).

For $\beta(Q^2)$ in the form (27) M-DAMA will generate an infinite number of the Q^2 poles concentrated near the "ionization point" $Q^2 = -Q_0^2$. Although these are in the unphysical region of negative Q^2, such a feature of the model
A) makes us think about $\beta(Q^2)$ as about a kind of trajectory, what is not the case, as it was stressed above, and
B) might create a problem for a general theoretical treatment, for example for making analytical continuation in Q^2. To avoid this we can redefine $\beta(Q^2)$ in the nonphysical Q^2 region, for example in the following way:

$$\beta(Q^2) = \begin{cases} -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q^2+Q_0^2}{Q_0^2}\right), & \text{for } Q^2 \geq 0\,, \\ -1 - \frac{\alpha_t(0)}{\ln g} \ln\left(\frac{Q_0^2-Q^2}{Q_0^2}\right), & \text{for } Q^2 < 0\,. \end{cases} \tag{34}$$

This function has a maximum at $Q^2 = 0$, $\beta(0) = -1$. M-DAMA with $\beta(Q^2)$ given by eq. (34) preserves all its good properties, discussed above, and does not contain any singularity in Q^2 (except for the supercritical Pomeron case, which we do not allow).

5 Conclusions

A new model for the Q^2-dependent dual amplitude with Mandelstam analyticity is proposed. The M-DAMA preserves all the attractive properties of DAMA, such as its pole structure and Regge asymptotics, but it also leads to generalized dual amplitude $A(s,t,Q^2)$ and in this way realizes a unified "two-dimensionally dual" picture of strong interaction [1, 2, 3, 4] (see Fig. 3). This amplitude, when $t = 0$, can be related to the nuclear SF, and in this way we fix the function $\beta(Q^2)$, which introduces the Q^2-dependence in M-DAMA, eq. (3). Our analyzes shows that for both large and low x limits as well as for the resonance region the results of M-DAMA are in qualitative agreement with the experiment.

In the proposed formulation a Q^2-dependence is introduced into DAMA through the additional function $\beta(Q^2)$. Although in the integrand this function stands next to Regge trajectories, this, as it was stressed already, does not mean that it also corresponds to some physical particles. There is no qualitative difference between two ways of introducing Q^2-dependence into DAMA: through the Q^2-dependent parameter g, i.e. function $g(Q^2)$ [1, 2] or through the function $\beta(Q^2)$. On the other hand the second way, i.e. M-DAMA, is applicable for all range of Q^2 and it results into physically correct behaviour in all tested limits.

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